The Fascination of Crystals and Symmetry

Unit 2.5

by Frank Hoffmann & Michael Sartor
Centered Cells

- If lattice points are only at the corners of the unit cell, it is a primitive lattice; there are 7 different primitive lattices.
- Addition of further lattice points – under retention of the symmetry – give rise to 7 more lattices, 7 centered lattices. This leads to the 14 Bravais lattices.

![Diagrams of primitive, single-side face-centered, body-centered, and all-side face-centered unit cells with labels P, C(AB), I, and F respectively.](image-url)
Centered Cells – formula units/lattice points

8 corners $\times \frac{1}{8} = 1$ lattice point/unit cell
Centered Cells – formula units/lattice points

\[ C(AB) \]

\[(8 \text{ corners } \times \frac{1}{8}) + (2 \text{ faces } \times \frac{1}{2}) = 2 \text{ lattice points/unit cell} \]
Centered Cells – formula units/lattice points

(8 corners x 1/8) + (1 inside) = 2 lattice points/unit cell
Centered Cells – formula units/lattice points

(8 corners \times \frac{1}{8}) + (6 faces \times \frac{1}{2}) = 4 \text{ lattice points/unit cell}
If lattice points are only at the corners of the unit cell, it is a primitive lattice; there are 7 different primitive lattices.

Addition of further lattice points – under retention of the symmetry – give rise to 7 more lattices, 7 centered lattices. This leads to the 14 Bravais lattices.
The 14 Bravais lattices

$7 \times 4 = 14$ (?)

crystal systems

Bravais lattices

kind of centerings
<table>
<thead>
<tr>
<th></th>
<th>triclinic</th>
<th>monoclinic</th>
<th>orthorhombic</th>
<th>tetragonal</th>
<th>hexagonal/trigonal</th>
<th>cubic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P</strong></td>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
<td><img src="image3.png" alt="Diagram" /></td>
<td><img src="image4.png" alt="Diagram" /></td>
<td><img src="image5.png" alt="Diagram" /></td>
<td><img src="image6.png" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>C(AB)</strong></td>
<td><img src="image7.png" alt="Diagram" /></td>
<td><img src="image8.png" alt="Diagram" /></td>
<td><img src="image9.png" alt="Diagram" /></td>
<td><img src="image10.png" alt="Diagram" /></td>
<td><img src="image11.png" alt="Diagram" /></td>
<td><img src="image12.png" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>I</strong></td>
<td><img src="image13.png" alt="Diagram" /></td>
<td><img src="image14.png" alt="Diagram" /></td>
<td><img src="image15.png" alt="Diagram" /></td>
<td><img src="image16.png" alt="Diagram" /></td>
<td><img src="image17.png" alt="Diagram" /></td>
<td><img src="image18.png" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>F</strong></td>
<td><img src="image19.png" alt="Diagram" /></td>
<td><img src="image20.png" alt="Diagram" /></td>
<td><img src="image21.png" alt="Diagram" /></td>
<td><img src="image22.png" alt="Diagram" /></td>
<td><img src="image23.png" alt="Diagram" /></td>
<td><img src="image24.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>
The 14 Bravais lattices

* 23th August 1811 in Annonay, Frankreich
† 30th March 1863 in Le Chesnay
French physicist, crystallographer, universal scholar

Auguste Bravais

in 1848 he could show that there are only
14 unique different lattice types in 3D space

→ some of the 28 conceivable lattice types are redundant
→ some of them are not possible due to symmetry reasons
The 14 Bravais lattices – example for redundancy

monoclinic

P

I

C
The 14 Bravais lattices – example for non-compatible symmetry

cubic

? \rightarrow C \quad a \rightarrow \frac{\sqrt{2}}{2} a

tetragonal P
The 14 Bravais lattices