Script Unit 3.1

Welcome to the first unit of Chapter 3!

Slide 2

In this chapter we will enter the beautiful world of symmetry more deeply and will discuss symmetry aspects of various kinds of macroscopic objects - this means we will look at the objects from the outside and do not consider their inner structure, which might have translational symmetry too, as is the case for instance for crystalline objects.

Slide 3

More general, we are on the way to classify crystal structures in a systematic and hierarchical way - and two steps on this step ladder are already accomplished:

We are now familiar with the 7 crystal systems and the 14 Bravais lattices!

In this chapter we will climb another step upwards and will introduce the 32 crystal classes, which are also called crystallographic point groups.

And then in chapter 4 we will bring both things together: on the one hand the symmetry of macroscopic objects and on the other hand their internal translational symmetry. This leads us to the 230 space groups - the complete description of the symmetry of crystalline objects. Then you will have the total overview, there is no more to add...

Slide 4

Let’s start…

Symmetry in the linguistic sense means something like regularity or also harmony. And it is indeed the fact, that symmetry is perceived by humans as something harmonic and beautiful, symmetrical arrangements and symmetric patterns are pleasant to our eyes.

You see a first case of symmetry here on this slide, although not a perfect one, in which the typographic alignment of the text is "centered", this means, symmetrically aligned along an axis in the middle. This is often used for the title of a work, and for poems and songs. Centered text with multiple lines is considered less readable but nice to look at anyway.

Before we see what kinds of different symmetries exist, we have to introduce an important formal aspect, and this is the difference between the two terms symmetry operation and symmetry element.
A symmetry operation is a geometrical reorganization or transformation, which maps an object onto itself - so this is what you do with an object.

Let us look again at the example of unit 2.7… here the symmetry operation is - rotating this cube around this axis by 90° degrees - and you cannot distinguish the result from the start configuration!

The symmetry element in contrast is the geometrical object (it can be a point, a line or a plane) on which the symmetry operation is carried out.

In this example we rotated the cube by 90° around this axis - this means this axis is the geometrical object on which this rotation was carried out - so, this axis is our symmetry element, here a 4-fold axis of rotation. This axis comprises all points which do not change their positions while carrying out the symmetry operation - this is meant by this slightly formal expression “invariant spatial points”.

You now also should have an idea, what the meaning of point symmetry is: point symmetry means, you carry out a symmetry operation, in which at least one point of the object is not moving around. And this is different to translational symmetry, in which translation operations are involved.

A last thing concerning symmetry elements is, that usually more than one symmetry operation can be carried out on one symmetry element - here, for instance, you can also rotate the cube by 180° - and then this line would be a 2-fold axis of rotation.

What different kinds of symmetry elements are there, if we consider only external symmetry of macroscopic objects?

Only these five! Identity, Mirror plane, Axis of rotation, Center of inversion, and the Rotoinversion axis.

The first symmetry element - Identity - is quickly dealt with: all objects, even the most asymmetric ones have at least this symmetry element - this seems to be a bit contradictory, that an asymmetric object contains a symmetry element. However, you can consider this also as a 1-fold axis of rotation - if you rotate an object by exactly 360° then you carried out a symmetry operation!

The symbol for identity is E and this identity operation was introduced by mathematicians for a
formal reason belonging to group theory, the identity operation is so to say the neutral element in group theory, which we will not discuss in detail!

Let’s move on - and talk about mirror symmetry.

**Slide 8**

All objects shown here have something in common - sure - they possess mirror symmetry, which is also called line symmetry, reflection symmetry or also bilateral symmetry. An object, which does not change upon reflection has mirror symmetry.

If you consider flat, 2D objects, then the symmetry element is called a mirror line, if you consider three-dimensional objects then it is a mirror plane. To indicate a mirror plane graphically use simply a solid line, as shown here. As a character symbol a ‘m’ is used - easily to memorize.

An object can have of course more than one mirror plane; look at the character H, there is one horizontal and one vertical mirror line or plane - the resulting symbol to characterize this would be mm - we will come back to so-called ‘unique’ mirror planes after we considered rotational symmetry.

There is one additional aspect of bilateral symmetry which I want to share with you in a small excursus.

**Slide 9**

Here you see a picture of Kelly George - she won a beauty competition in the USA and hold the title Miss Arkansas in 2007…a beautiful face, of course. Normally, symmetric faces are being recognized as particularly beautiful.

To which extent a face actually is mirror symmetric can be easily explored by taking a photo and to mirror one half of the face and then glue the two parts together - this can be done either with the right- or left-hand side of this portrait - and this is the result of the two exactly mirror symmetric Kelly Georges!

This is interesting: we see, that Kelly has indeed a very symmetrical face - because the two versions differ not by to a large extent - but, if you asked 10.000 people which of these three pictures shows the most beautiful face - the majority would answer: it is the original one on the left side!

A slight asymmetry is preferred over a perfect mirror symmetry, as studies have found out!

This is one of a bunch of interesting results of anthropologists which explore the scientific area of attractiveness and the determining factors concerning the choice of partners!
One very active researcher on this field is Prof. Dr. Karl Grammer - you can find very interesting books and articles about his research on various platforms… and not only about his investigations on the perception of beauty.